PHYS4150 — PLASMA PHYSICS

LECTURE 11 -

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# 1 FLUID DESCRIPTION OF PLASMAS

## 1.1 *Fluid variables*

We first define the volume of a fluid parcel as

$$\mathrm{d}x^3 = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z,$$

which has the mass

$$m = (n_i m_i + n_e m_e)\,\mathrm{d}x^3$$

and the mass density

$$\rho = \frac{m}{\mathrm{d}x^3} = (n_i m_i + n_e m_e) = \rho_i + \rho_e.$$

Mass $m$ and density $\rho$ are fluid variables. We now introduce the average velocity $\mathbf{u}$ of the plasma particles in the fluid parcel. Obviously, the fluid parcel will flow at this speed. The distribution $f(\mathbf{v}|\mathbf{u})$ of particle speeds can often described by a shifted Maxwellian, i.e.

$$f(\mathbf{v}|\mathbf{u}) = \frac{n}{(\pi v_{th}^2)^{3/2}} \exp\left\{-\frac{(\mathbf{v}-\mathbf{u})^2}{v_{th}^2}\right\}.$$

Knowledge of $f(\mathbf{v})$ allows us to compute $\mathbf{u}$

$$\mathbf{u} = \frac{\int \mathbf{v} f(\mathbf{v})\,\mathrm{d}x^3}{\int f(\mathbf{v})\,\mathrm{d}x^3} = \frac{1}{n}\int \mathbf{v} f(\mathbf{v})\,\mathrm{d}x^3.$$

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The next fluid variable to discuss is the particle flux through a face of the fluid volume $\mathrm{d}x^3$. Let us consider the flux through the $\mathrm{d}x\,\mathrm{d}y$ face. Because flux is number of particles per area and time we can write

$$\Gamma_{xy} = \frac{N}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t},$$

or after multiplying with $\mathrm{d}z/\,\mathrm{d}z$

$$\Gamma_{xy} = \frac{N}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z}\frac{\mathrm{d}z}{\mathrm{d}t} = nu_z,$$

where $u_z$ is the z component of fluid velocity. In general

$$\bar{\Gamma} = n\mathbf{u}.$$

The current density $\mathbf{j}$ of the fluid parcel is the charge flux

$$\mathbf{j} = nq\mathbf{u}.$$

The last fluid variable we need to consider is the pressure, which in fact is the flux of momentum evaluated in the frame moving with $\mathbf{u}$.

## 1.2 *Continuity equation*

$$\frac{\partial}{\partial t}m = \frac{\partial}{\partial t}(\rho\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z) = \sum \text{inward-flow} \;= \rho u_x\,\mathrm{d}y\,\mathrm{d}z$$

6 sides:

$$\int \frac{\partial\rho}{\partial t}\,\mathrm{d}x^3 = -\oint \rho\mathbf{u}\,\mathrm{d}A$$

divergence theorem: $\oint \mathbf{F}\,\mathrm{d}A = \int(\nabla\mathbf{F})\,\mathrm{d}x^3$:

$$\int \frac{\partial\rho}{\partial t}\,\mathrm{d}x^3 = -\int \nabla(\rho\mathbf{u})\,\mathrm{d}x^3$$

or

$$\int \left[\frac{\partial\rho}{\partial t} + \nabla(\rho\mathbf{u})\right]\mathrm{d}x^3 = 0$$

Continuity equation:

$$\boxed{\frac{\partial\rho}{\partial t} + \nabla(\rho\mathbf{u}) = 0}$$

In general, if **H** is conserved, then

$$\frac{\partial}{\partial t}\mathbf{H} + \nabla(\mathbf{Hu}) = 0,$$

where $(\mathbf{Hu})$ is the flux of **H**. If there is a source term $S = \frac{\text{new mass}}{\mathrm{d}x^3}$, then we need to change the continuity equation to

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla(\rho\mathbf{u}) = S.}$$